7.1 Integral As Net Change

EXAMPLE 1 Interpreting a Velocity Function

Figure 7.1 shows the velocity

$$\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2} \qquad \frac{\text{cm}}{\text{sec}}$$

of a particle moving along a horizontal s-axis for $0 \le t \le 5$. Describe the motion.

Solve Graphically

moving left $0 \le t < 1.255$

stops at $t = 1.255$

moving right $1.253 < t \le 5$

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at (a) $t = 1$ sec? (b) $t = 5$ sec?

$8(t+1)^{-2}$

a) $\underbrace{\int_0^1 \left(t^2 - \frac{8}{(t+1)^2}\right) dt}_{\text{change in position from } t=0 \text{ to } t=1} + 9$

$\frac{1}{3}t^3 - (-8)(t+1)^{-1} \Big|_0^1 + 9$

$\left[\frac{1}{3} + \frac{8}{2}\right] - \left[0 + 8\right]$

$\frac{13}{3} - 8 + 9$

$\frac{13}{3} - \frac{24}{3} + \frac{27}{3} = \frac{16}{3}$

EXPLORATION 1 Revisiting Example 2

The velocity of a particle moving along a horizontal s-axis for $0 \le t \le 5$ is

$$\frac{ds}{dt} = t^2 - \frac{8}{(t+1)^2}.$$

1. Use the indefinite integral of ds/dt to find the solution of the initial value problem

$$\frac{ds}{dt} = t^2 - \frac{8}{(t+1)^2}, \qquad s(0) = 9.$$

2. Determine the position of the particle at $t = 1$. Compare your answer with the answer to Example 2a.
3. Determine the position of the particle at $t = 5$. Compare your answer with the answer to Example 2b.

$$\int ds = \int t^2 - \frac{8}{(t+1)^2}\,dt$$

$$S = \frac{1}{3}t^3 + \frac{8}{t+1} + C$$

$$9 = \frac{1}{3}(0) + \frac{8}{0+1} + C$$

$$C = 1$$

$$S = \frac{1}{3}t^3 + \frac{8}{t+1} + 1$$

EXAMPLE 3 Calculating Total Distance Traveled

Find the *total distance traveled* by the particle in Example 1.

math 9 $\int_0^5 \left| t^2 - \frac{8}{(t+1)^2} \right| dt$

Strategy for Modeling with Integrals

1. *Approximate what you want to find as a Riemann sum* of values of a continuous function multiplied by interval lengths. If $f(x)$ is the function and $[a, b]$ the interval, and you partition the interval into subintervals of length Δx, the approximating sums will have the form $\sum f(c_k)\,\Delta x$ with c_k a point in the kth subinterval.
2. *Write a definite integral,* here $\int_a^b f(x)\,dx$, to express the limit of these sums as the norms of the partitions go to zero.
3. *Evaluate the integral* numerically or with an antiderivative.

EXAMPLE 4 Modeling the Effects of Acceleration

A car moving with initial velocity of 5 mph accelerates at the rate of $a(t) = 2.4t$ mph per second for 8 seconds.

(a) How fast is the car going when the 8 seconds are up?

(b) How far did the car travel during those 8 seconds?

a) $a(t) = 2.4t$

$$\int_0^8 2.4t\,dt + 5$$

$$1.2t^2 \Big|_0^8 + 5$$

$$1.2(8)^2 + 5 = 81.8 \text{ mph}$$

b) $v(t) = 1.2t^2 + 5$

$$\int_0^8 (1.2t^2 + 5)\,dt$$

$$= .4t^3 + 5t \Big|_0^8$$

244.8 mph × seconds

$$244.8 \text{ mph} \times \frac{\text{second}}{3600 \text{ sec}} \cdot h$$

$= 0.068$ mi

EXAMPLE 5 **Potato Consumption**

From 1970 to 1980, the rate of potato consumption in a particular country was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

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EXAMPLE 6 Finding Gallons Pumped from Rate Data

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in Table 7.1. How many gallons were pumped during that hour?

Table 7.1 Pumping Rates

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

March 10, 2016

Work

In everyday life, *work* means an activity that requires muscular or mental effort. In science, the term refers specifically to a force acting on a body and the body's subsequent displacement. When a body moves a distance d along a straight line as a result of the action of a force of constant magnitude F in the direction of motion, the **work** done by the force is

$$W = Fd.$$

The equation $W = Fd$ is the **constant-force formula** for work.

The units of work are force $\times$ distance. In the metric system, the unit is the newton-meter, which, for historical reasons, is called a joule (see margin note). In the U.S. customary system, the most common unit of work is the **foot-pound.**

Hooke's Law for springs says that the force it takes to stretch or compress a spring x units from its natural (unstressed) length is a constant times x. In symbols,

$$F = kx,$$

where k, measured in force units per unit length, is a characteristic of the spring called the **force constant.**

EXAMPLE 7 A Bit of Work

It takes a force of 10 N to stretch a spring 2 m beyond its natural length. How much work is done in stretching the spring 4 m from its natural length?